

Meshless Methods for Solid Mechanics in *Mathematica**

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May 19, 2010

Abstract

Meshless methods are emerging techniques for the numerical solution of problems in the fields of solid mechanics, fluid-dynamics and, in general, partial differential equations (PDE) with boundary conditions (see [1] for a comprehensive introduction and [2, 3, 4, 5] for some of the last developments). Briefly, a meshless method consists in discretizing the problem-specific equations using a set of scattered points (nodes) inside the integration domain, instead of using an element-based mesh discretization as in the finite-element method (FEM).

One of the main differences between the FEM and a Meshless method is the definition of the shape functions. In the former, shape functions are element-based and depend only on the nodal values of the current element. In the latter, instead, since there are no elements, the shape functions at a given point depend on some basis functions and on a set of neighbor nodes of the current point itself.

Once the shape functions are defined, the PDE problem is discretized in a similar fashion to the FEM approach thus involving both surface and volume integrals to be numerically solved. As a result the discretized PDE problem is reformulated as a linear system to be solved. For instance, in a problem of linear elasticity, the PDE problem becomes

$$\mathbf{K}\mathbf{x} = \mathbf{f}$$

where \mathbf{K} is the stiffness matrix, \mathbf{x} and \mathbf{f} are the unknown nodal displacement and the force vector respectively.

The research in the field of meshless methods is relatively new and then several problems still have to be solved. Currently there is not a widely-used commercial software that implements these methods and hence no extensive test campaigns can be run.

The main goal of the present work is to provide an easy-to-use *Mathematica* package that implements the basic features of some existing meshless methods for the simulation of solid elasticity in both 2D and 3D cases. The package is implemented in an *object-oriented* fashion and provides the user with the following features:

*Submitted to the *4th Mathematica Italia User Group Meeting 2010*.

- a set of different basis functions;
- a set of different approximation schemes;
- a tool for the numerical integration over generic 1D, 2D and 3D domains;
- several functions to set the boundary conditions and the specific constraints of the problem;
- several graphics utilities for the visualization of the results.

Some test cases have been implemented to show the capabilities of the meshless methods. Taking advantage of the *Mathematica* computational and graphical capabilities and of the modularity of the implementation, the following aspects have been easily inspected:

- comparison of different approximation schemes in terms of precision and computational cost;
- effects of different domain discretizations using regularly- or irregularly-spaced nodes;
- comparison of different meshless methods for solid mechanics.

References

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